Slope Safety Factor Analysis Using ANSYS

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Abstract

This paper presents an analysis method for slope safety factor through soil shear strength reduction algorithm using finite elements. When the system reaches instability, the numerical non-convergence occurs simultaneously. The safety factor is obtained by strength reduction algorithm. The numerical convergence or non-convergence is related to the yield criterion. This paper presents an analysis and comparison of several yield criterions in common use and deduced the substitutive relationship between them. For convenience the Mohr-Coulomb criterion is replaced by Mohr-Coulomb equivalent area circle criterion, which was proposed by professor Xu Gancheng and Zheng Yingren in 1990. Through a series of case studies, the safety factor of FEM is fairly close to the result of traditional limit equilibrium method. The applicability of the proposed method was clearly exhibited. It suited to the complicated geological condition and supported slope. No assumption needs to be made in advance about the shape or location of the failure surface. It is a promising method.

Introduction

Today, the traditional method of slope stability analysis mainly include limit-equilibrium, limit analysis, slip-line etc. These methods based on the theory of limit-equilibrium cannot involve the stress-strain behavior of soil and need assumptions of failure surface shape (circular, log-spiral, piecewise linear, etc) in advance. It typically restricted to Mohr-Coulomb soil models. The FEM method has many advantages over the traditional method. It is not only satisfies the equilibrium condition of stress, but also involves the stress-strain relation. The result is more reliable. The traditional FEM method just computes the stress field, displacement field and plastic zone. It cannot get the slope stability safety factor. With the development of computer techniques and the theory of generalized plastic mechanics of soil, the FEM Program (such as ANSYS) make great progress in nonlinear finite element techniques. Its preprocessing and post-processing become more and more convenient. In this paper the ANSYS (R) Release 5.6.1was used to analysis the stability of slope.

This paper presented an analysis method for slope safety factor through shear strength reduction algorithm by ANSYS. The safety factor of slope is defined as the factor by which the original shear strength parameters be reduced to bring the slope to the point of failure. When the system reaches instability, the numerical non-convergence occurs simultaneously. The safety factor is obtained by shear strength reduction algorithm. The safety factor is related to the yield criterion. This paper discussed several yield criterions in common use and deduced the substitutive relationship between them. For convenience the Mohr-Coulomb criterion is replaced by Mohr-Coulomb equivalent area circle criterion, which was proposed by professor Xu Gancheng and Zheng Yiren in 1990. Finally, the strength reduction technique is illustrated through a number of examples.

The advantage of the FEM method over traditional method

1) No assumption needs to be made in advance about the shape or location of the failure surface.

2) It can involve the non-linear elastic-plastic model, such as Mohr-Coulomb, Von Mises and Drucker-Prager model etc.

3) Most importantly, the critical failure surface is found automatically. It can monitor progressive failure up to and including overall shear failure. The nature soil slope for 2D plane strain mesh is shown in figure 1. Figure 2 shows the deformed mesh. Figure 3 shows the continuous contours of X displacement. The failure surface is shown in Figure 4. Figure 5 shows the velocity field at limit state.

4) It can simulate the interaction between soil and support, such as pile and anchor etc. figure 6 shows the plastic strain zone without supporting (anchor element was killed). Figure 7 shows the plastic strain zone with supporting of anchor (anchor element was activated). From figure 6 we can see the failure surface is irregular. The failure surface moved to the end of the anchor.

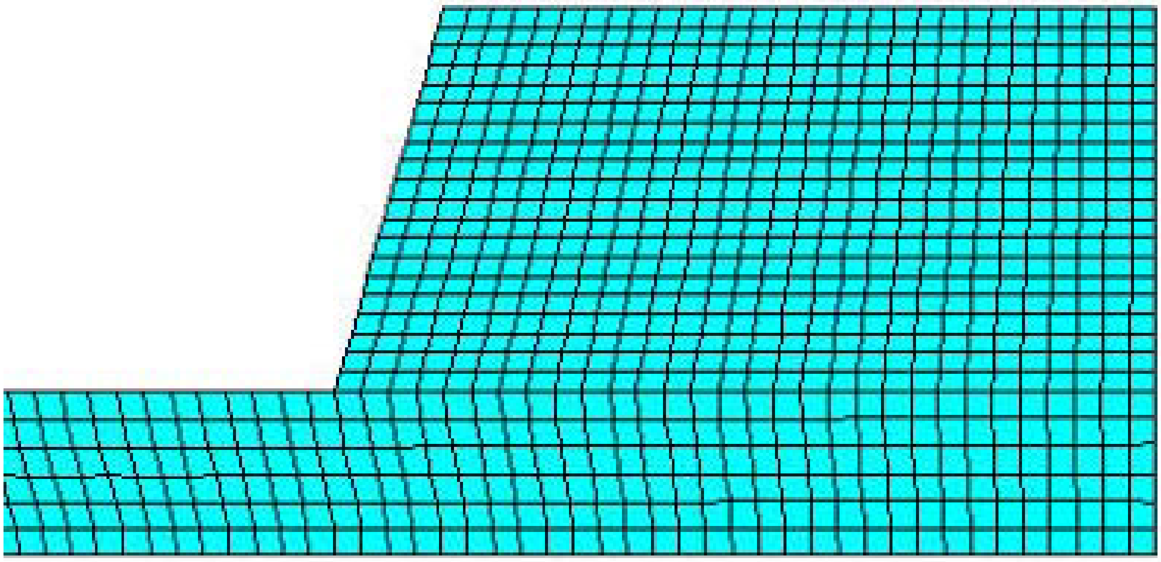

Figure 1 - Finite element mesh for nature slope

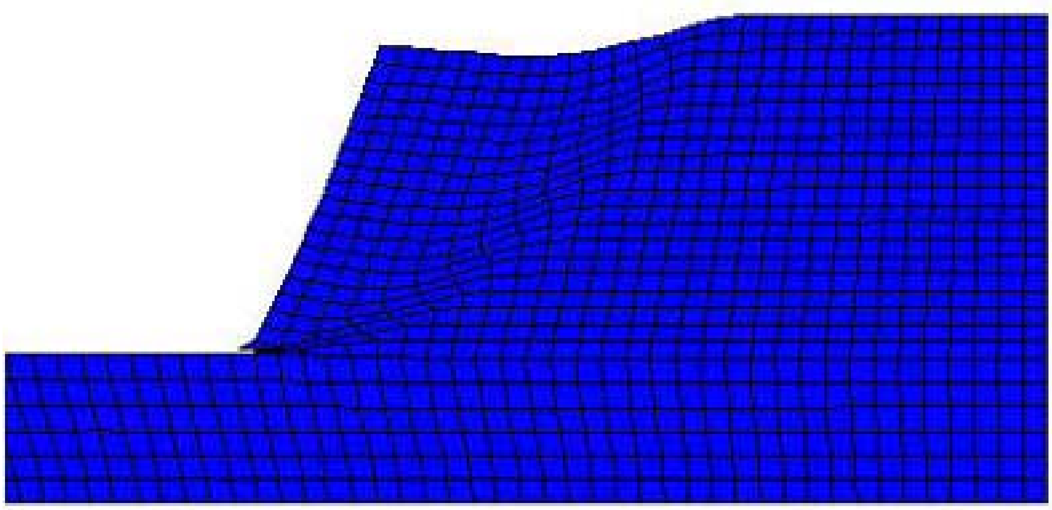

Figure 2 - Deformed mesh

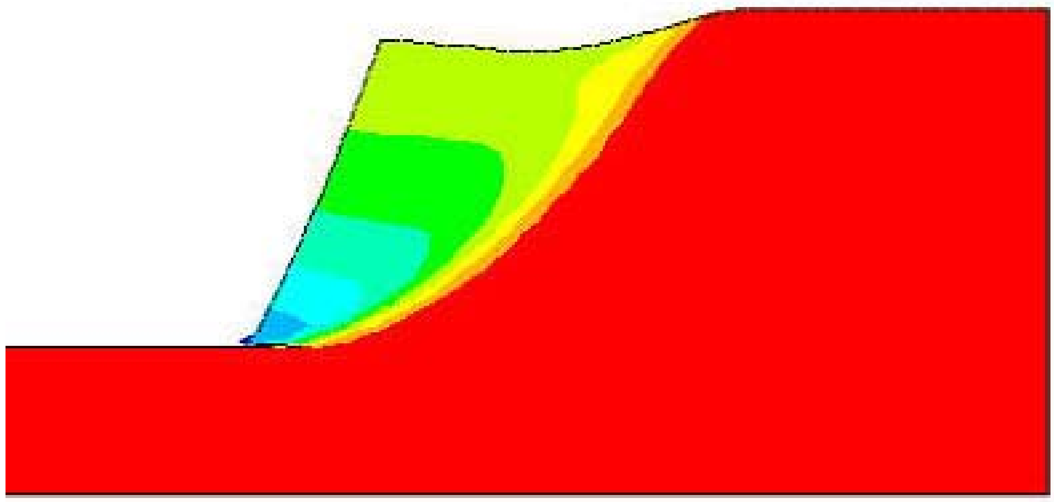

Figure 3 - Continuous contours of X displacement

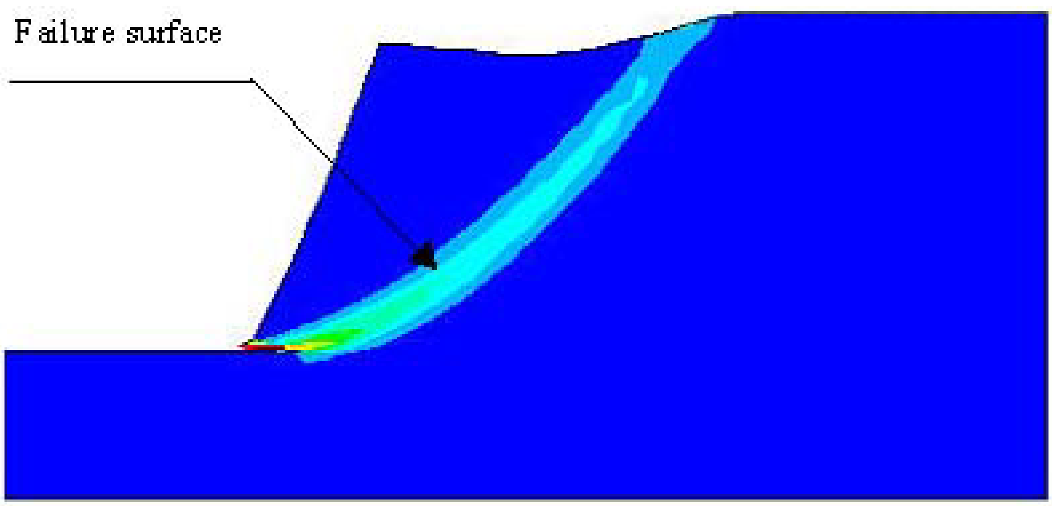

Figure 4 - Continuous contours of equivalent plastic strain

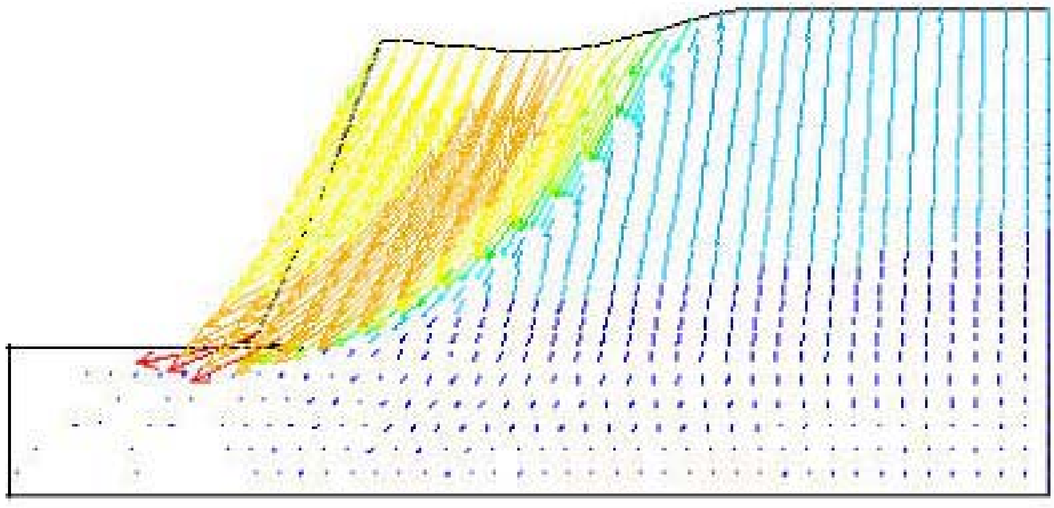

Figure 5 - Velocity field at the limit state

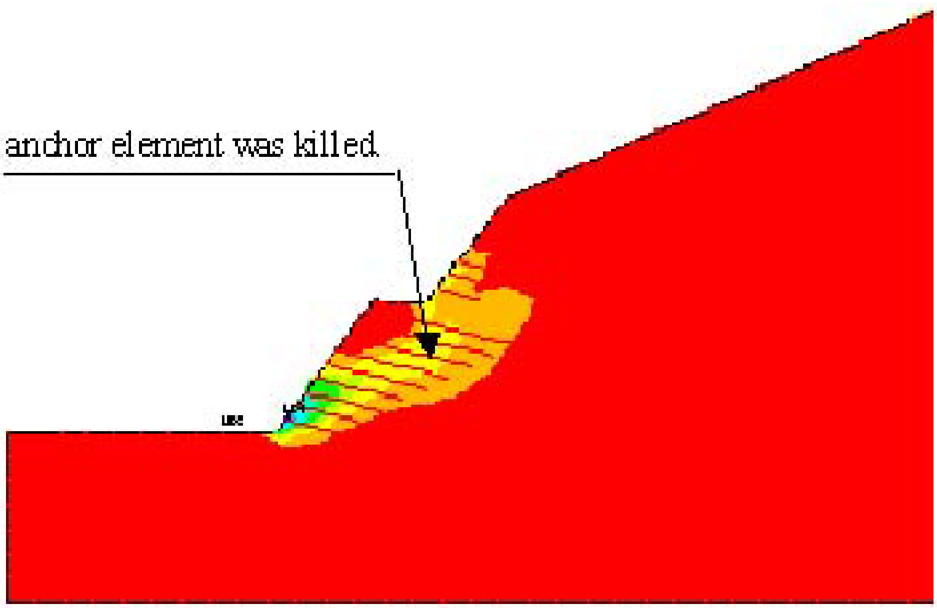

Figure 6 - The plastic strain zone without supporting

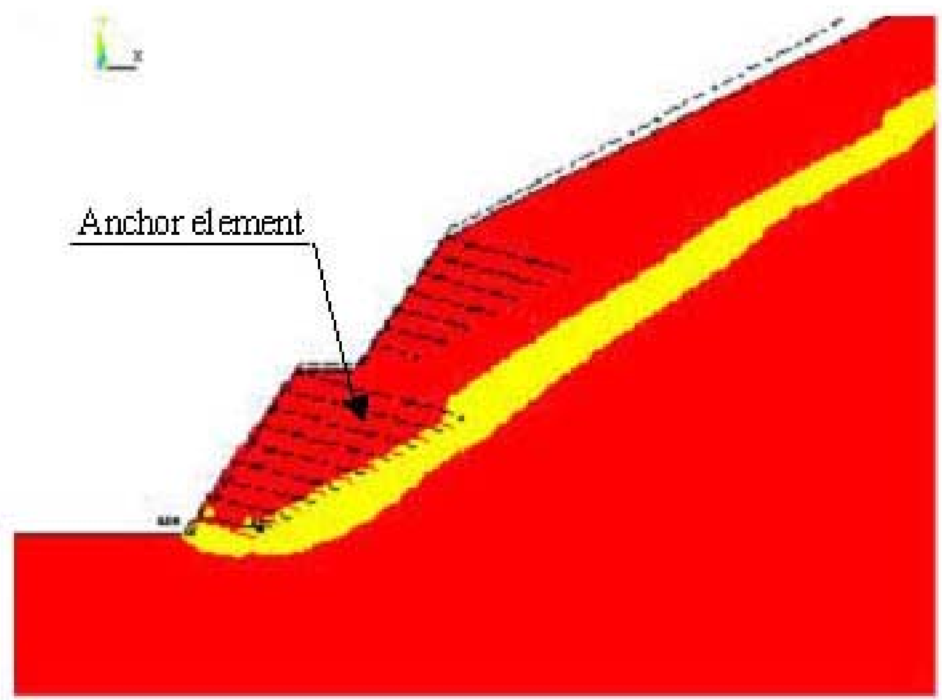

Figure 7 - The plastic strain zone with supporting of anchor

The definition of the safety factor for slope stability

In order to assess the factor of safety, we introduced a reduction coefficient SF in the Drucker-Prager yield model. The Drucker-Prager yield function is expressed with the strength reduction coefficient SF as

$$F = \frac{\alpha}{SF} I_1 + \sqrt{J_2} = \frac{k}{SF} \qquad (1)$$

where: the variable of SF gives the factor of safety when the slope attains at the limit state to undergo failure.

The shear strength reduction algorithm proceeds in steps is described in following table.

1	Preprocessing		Creating geometrical mesh, defined material properties, generating a finite element mesh, Apply Loads.
2	Initial value i is the iteration counter , SF^i the value of the safety of factor, α , k the function of cohesion c and friction angle ϕ .		i=1
			$SF^i = 1$
			$\alpha^i = \alpha$
			$k^i = k$
3	Solve the problem.		
4	Check failure	If the instability is reached	$SF = SF^{(i-1)}$, go to end
		If the instability is not reached	i=i+1

		solve the equation (a) and (b), get the value of cohesion C and angle of friction ϕ as input data.	$SF^i = SF^{(i-1)} + \Delta f$
			$\alpha^i = \dfrac{\alpha}{SF^i} \qquad (a)$
			$k^i = \dfrac{k}{SF^i} \qquad (b)$
			Go to 2
4	End		
5	Post processing		

Yield criterions in common use

The constitutive law for soil material includes elasticity and plasticity model like Mohr-Coulomb, Von Mises and Drucker-Prager etc. These models only require the elastic constant E (Young's modulus), γ (Poisson's ratio), the cohesion C and friction angle ϕ. The Drucker-Prager criterion is more convenient from of view of numerical efficiency.

The Drucker-Prager yield function is expressed as

$$f(\sigma) = \alpha I_1 + \sqrt{J_2} = k \qquad (2)$$

I_1 and J_2 are stress invariants and constants α, k can be defined from common geotechnical data: cohesion C and angle of friction ϕ.

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3, \qquad J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2\right]$$

When α and k are defined as

$$\alpha = \frac{2\sin\phi}{\sqrt{3}(3 - \sin\phi)}, \quad k = \frac{6c\cos\phi}{\sqrt{3(3 - \sin\phi)}} \qquad (3)$$

the yield surface on the deviatoric plane (π plane) is circumcircle of hexagon. The hexagon with incoordinate angle is the yield surface of Mohr-Coulomb criterion on the deviatoric plane. Figure 8 shows the yield surface on the deviatoric plane.

When the α and k are defined as

$$\alpha = \frac{\sin\phi}{\sqrt{3}\sqrt{3 + \sin^2\phi}}, \quad k = \frac{\sqrt{3}c\cos\phi}{\sqrt{3 + \sin^2\phi}} \qquad (4)$$

the yield surface on the deviatoric plane is inscribed circle of hexagon.

When the α and k are defined as

$$\alpha = \frac{2\sqrt{3}\sin\phi}{\sqrt{2\sqrt{3}\pi(9 - \sin^2\phi)}}, \quad k = \frac{6\sqrt{3}c\cos\phi}{\sqrt{2\sqrt{3}\pi(9 - \sin^2\phi)}} \qquad (5)$$

the yield surface on the deviatoric plane is the equivalent area circle of the hexagon. It is called Mohr-Coulomb equivalent area circle criterion. It was proposed by professor Xu Gancheng and Zheng Yiren in

1990. A cone characterizes the Mohr-Coulomb surface in three-dimensional stress space with the vertices in deviatoric cross section. It brings difficult to numerical analysis. For convenience this surface is replaced by a smooth surface yield criterion -- Mohr-Coulomb equivalent area circle criterion.

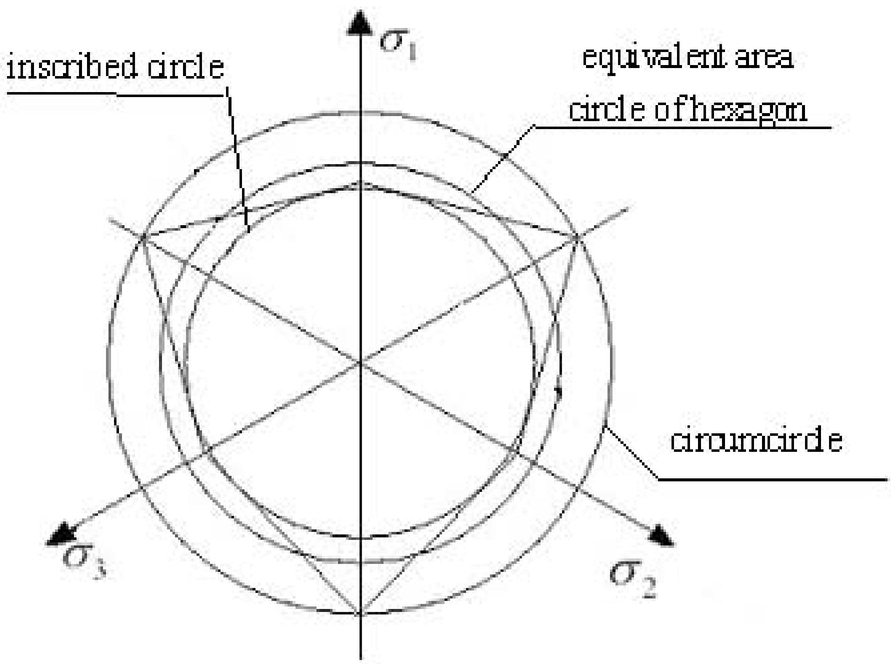

Figure 8 - The yield surface on the deviatoric plane

The substituent relationship between the criterions.

The safety factor is related to the yield criterion. With different yield criterion, we get different safety factor. But it can be substituted by each other. Take equation 3 and 5 for example:

$$f_1 = \sqrt{J_2} = -\alpha_1 I_1 + k_1$$

$$\alpha_1 = \frac{2\sin\phi}{\sqrt{3}(3-\sin\phi)} \quad , \quad k_1 = \frac{6c\cos\phi}{\sqrt{3}(3-\sin\phi)}$$

$$f_2 = \sqrt{J_2} = -\alpha_2 I_1 + k_2$$

$$\alpha_2 = \frac{2\sqrt{3}\sin\phi}{\sqrt{2\sqrt{3}\pi(9-\sin^2\phi)}} \quad , \quad k_2 = \frac{6\sqrt{3}c\cos\phi}{\sqrt{2\sqrt{3}\pi(9-\sin^2\phi)}}$$

$$\because \quad \eta = \frac{\alpha_1}{\alpha_2} = \frac{k_1}{k_2} = \sqrt{\frac{2\pi}{3\sqrt{3}} \times \frac{3+\sin\varphi}{3-\sin\varphi}} = f(\phi)$$

$$\therefore \quad \alpha_1 = \eta\alpha_2 \, , \quad k_1 = \eta k_2$$

$$\therefore \quad f_1 = -\alpha_1 I_1 + k_1 = -\eta\alpha_2 I_1 + \eta k_2 = \eta(-\alpha_2 I_1 + k_2)$$

$$\therefore \frac{f_1}{f_2} = \frac{\eta(-\alpha_2 I_1 + k_2)}{-\alpha_2 I_1 + k_2} = \eta = \sqrt{\frac{2\pi}{3\sqrt{3}} \times \frac{3+\sin\varphi}{3-\sin\varphi}}$$

With different value of ϕ, we get different value of η as follow

ϕ (°)	0	10	20	30	40	50	60	70	80	90
η	1.10	1.165	1.233	1.301	1.367	1.428	1.480	1.521	1.546	1.555

The Drucker-Prager criterion is defined by equation (3) in ANSYS. When we get the value of η we can transform the safety factor under Drucker-Prager criterion to the factor under Mohr-Coulomb equivalent area circle criterion.

Case study

Soil slope, the height of slope $H = 20m$, unit weight of soil $\gamma = 25kN/m^3$, Young's modulus $E = 10MPa$, Poisson's ratio $\gamma = 0.2$, cohesion $C = 42kPa$, friction angle $\phi = 17^\circ$, slope angle $\beta = 30°、35°、40°、45°、50°$. The result is shown in following table.

Slope angle β		30°	35°	40°	45°	50°
FEM method	D-P criterion of circumcircle of hexagon (equation 3).	1.78	1.62	1.48	1.36	1.29
	Mohr-Coulomb equivalent area circle criterion (equation 5).	1.47	1.34	1.22	1.12	1.06
Conventional methods	Simplified Bishop	1.394	1.259	1.15	1.06	0.99
	Spencer	1.46	1.32	1.21	1.12	1.04

From this table we can see the obtained factor of D-P criterion of circumcircle of hexagon (equation 3) is proportional to the factor of Mohr-Coulomb equivalent area circle criterion (equation 5). The ratio is equal to η. The factor with Mohr-Coulomb equivalent area circle criterion is fairly close to the result of traditional limit equilibrium method with Mohr-Coulomb criterion (Spencer's method). In addition we found that the result depend on the accuracy of employed finite element mesh and the boundary conditions. If the mesh is too coarse the result will be not accurate enough.

Conclusion

1) The traditional limit-equilibrium method is still the most common way to analysis slope stability. But it needs assumption of the failure surface shape and location. It cannot involve the stress-strain behavior of soil. When the slope is supported with anchor or pile the traditional method cannot be used for this problems. In contrast, the strength reduction technique can be used. It is able to deal with more complicated slope retaining anchors and piles.

2) The strength reduction technique gives a factor of safety with respect to soil shear strength. In fact this is the traditional definition of safety factor for slope stability. Through shear strength reduction algorithm by finite elements, the critical failure surface is found automatically. It can monitor progressive failure up to and including overall shear failure.

3) The result depends on the accuracy of employed finite element mesh, boundary conditions and the convergence criterion. So the finite element model includes mesh, boundary condition, nonlinear convergence criteria must satisfy the accuracy requirement.

4) The obtained safety factor is related to the employed yield criterion. We found there is proportional relationship between the obtained factors with different yield criterion.

5) Mohr-Coulomb criterion is still the most common way. But its yield surface on the deviatoric plane is a hexagon with incoordinate angle. It brings difficulty to numerical analysis. For convenience the Mohr-Coulomb criterion is replaced by Mohr-Coulomb equivalent area circle criterion, which was proposed by professor Xu Gancheng and Zheng Yingren in 1990. Through a series of case studies, the safety factor of FEM with Mohr-Coulomb equivalent area circle criterion is fairly close to the result of traditional limit equilibrium method (Spencer's method). The applicability of the proposed method was clearly exhibited. It is a promising method.

6) We can also use this method to analysis the active or passive earth pressure and the stability of tunnels.

References

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